1957

Investigation of the application of Sutcliffe's development formula for forecasting deepening or filling and speed of cyclonic systems.

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by

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
AEROLOGY

United States Naval Postgraduate School
Monterey, California

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This work is accepted as fulfilling the thesis requirements for the degree of

MASTER OF SCIENCE

IN

AEROLOGY

from the

United States Naval Postgraduate School

ABSTRACT

The terms of Sutcliffe's development formula are evaluated by finite differences for one hundred cyclones. The values of relative divergence obtained are correlated with the subsequent 24-hour pressure change and 24-hour average speed. A correlation of -.08 was obtained between the relative divergence at the center and the subsequent 24-hour pressure change. Computing the algebraic sum of relative divergence across the center and correlating this value with the 24-hour pressure change gave a coefficient of .28. Using the algebraic difference of relative divergence across the center and correlating this with the subsequent 24-hour average speed gave a coefficient of -.02. It therefore appears that the finite difference evaluation of the Sutcliffe's development equation, as made in this investigation, is not a useful method for a 24-hour prognosis of the deepening, filling, or movement of winter cyclones.

The author wishes to express gratitude for the assistance of Professor G. J. Haltiner, Department of Aerology, U. S. Naval Postgraduate School, Monterey, California.

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f	Coriolis Parameter	
ζ	Scalar Relative Vorticity	$\mathbb{k} \cdot (\nabla \times \mathbb{V})$
ζ_g	Scalar Relative Geostrophic Vorticity	$\mathbb{k} \cdot (\nabla \times \mathbb{V}_g)$
ζ_T	Scalar Relative Thermal Vorticity	$\mathbb{k} \cdot (\nabla \times \mathbb{V}_T)$
$\mathbb{V}$	Wind Velocity	
$\mathbb{V}_g$	Geostrophic Wind Velocity	
$\mathbb{V}_H$	Horizontal Wind Velocity	
$\mathbb{V}_T$	Thermal Wind Velocity	
V_T	Scalar Thermal Wind	
∇	"del" Operator	$\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbb{k}\frac{\partial}{\partial z}$
∇^2	Laplacian Operator	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

1. Introduction.

Any attempt to forecast the future position or intensity of a cyclonic center should be based on the present dynamic processes as well as the changes which these processes will undergo. The latter part of this statement involves the more difficult phase of forecasting. There have been various approaches to the solution of this problem, none have been one hundred percent effective. In some instances the forecast is entirely subjective and depends primarily on the experience of the forecaster. On the other hand, empirical or statistical rules have been devised which give satisfactory results in some geographical areas. It was the purpose of this investigation to see if Sutcliffe's development formula [3] would serve as an accurate indication of the future intensity of a cyclonic center and the average speed it would maintain for a twenty-four hour period.

2. Resumé of the derivation of Sutcliffe's development formula.

It may be desirable to briefly outline the derivation of this development formula and note the assumptions that are made. Essentially he begins with the general vorticity equation in the form

$$\frac{\partial \zeta}{\partial t} + \mathbb{V}\cdot\nabla_p(\zeta+f) = -(\zeta+f)(\nabla_p\cdot\mathbb{V}) - \mathbb{k}\cdot\left(\nabla_p\,\omega \times \frac{\partial \mathbb{V}}{\partial p}\right) \qquad (1)$$

where pressure is taken as the vertical variable. According to estimates by Sutcliffe, the last term, called the "tilt" term, is largely offset by $\zeta(\nabla_p\cdot\mathbb{V})$. Another modification is made by neglecting the vertical advection of vorticity which is normally believed to be small. These simplifications reduce equation (1) to

$$\frac{\partial \zeta}{\partial t} + \mathbb{V}_H\cdot\nabla_p(\zeta+f) = -f(\nabla_p\cdot\mathbb{V}_H) \;. \qquad (2)$$

Equation (2) involves the divergence at any one level, but the local field of pressure is not the result of the motion at one level, but rather of the integrated effects of the three dimensional motion. In order to get an estimate of the contribution of a layer, the difference is taken, resulting in

$$f(\nabla_p\cdot\mathbb{V}_H - \nabla_p\cdot\mathbb{V}_{Ho}) = \frac{\partial(\zeta_o-\zeta)}{\partial t} + \mathbb{V}_{Ho}\cdot\nabla_p(\zeta_o+f) - \mathbb{V}_H\cdot\nabla_p(\zeta+f) \qquad (3)$$

where the subscript "O" denotes the lower layer. Applying the geostrophic approximation for vorticity in the form of

$$\zeta = \frac{g}{f}\,\nabla_p^2 z$$

the local rate of change of the difference in vorticity becomes

$$-\frac{g}{f}\frac{\partial}{\partial t}\left(\nabla_p^2 z - \nabla_p^2 z_0\right) = -\frac{g}{f}\nabla_p^2\frac{\partial h}{\partial t} \qquad (4)$$

where "h" is the thickness between the two levels. Substitution of equation (4) into equation (3) gives

$$f(\nabla_p \cdot \mathbb{V}_H - \nabla_p \cdot \mathbb{V}_{H0}) = -\frac{g}{f}\nabla_p^2\frac{\partial h}{\partial t} + \mathbb{V}_{H0}\cdot\nabla_p(\rho_0 + f) - \mathbb{V}_H \cdot \nabla_p(\rho + f). \qquad (5)$$

In order to use this equation as a prognostic tool, the term of $\frac{\partial h}{\partial t}$ must be estimated or replaced by some reasonable assumption for the thickness change. In this derivation it is assumed that temperature changes and therefore thickness changes are purely advective; and, finally, that this advection may be approximated by the geostrophic wind. That is $\frac{\partial h}{\partial t} = -\mathbb{V}_g \cdot \nabla h$. With this assumption, non-adiabatic effects and vertical motions are neglected, and in some areas these may have marked affects on the thermal field. Using the advective assumption, as well as the equations, $\mathbb{V}_g = \mathbb{V}_0 + \mathbb{V}_T$ and $\rho_g = \rho_{g0} + \rho_T$, the final form of equation (5) gives the "relative divergence" of Sutcliffe as

$$(\nabla_p \cdot \mathbb{V}_H - \nabla_p \cdot \mathbb{V}_{H0}) = -\frac{V_T}{f}\Big(\underset{(a)}{2\frac{\partial \rho_0}{\partial s'}} + \underset{(b)}{\frac{\partial \rho_T}{\partial s'}} + \underset{(c)}{\frac{\partial f}{\partial s'}}\Big) \qquad (6)$$

where s' is the arc length in the direction of the thickness lines.

We will now examine each of the various terms of this equation.

a. From term (a) it is indicated that thermal wind over

a surface center of positive vorticity implies relative divergence ahead, relative convergence to the rear. However, this term has no effect on the intensification of the center, because at the center $\frac{\partial \rho_0}{\partial s'}$ is zero. Hence it merely affects the propagation of the wave.

b. $-\frac{V_T}{f}\frac{\partial \rho_T}{\partial s'}$ This term indicates that there should be relative divergence where the thermal vorticity decreases in the direction of the thermal wind and relative convergence where the thermal vorticity increases in the direction of wind shear. In the normal synoptic situation, where the thickness trough is in the rear of a surface depression and a thickness ridge over or ahead of it, there would be a decrease of thermal vorticity downstream over the surface depression, which would mean relative divergence. On the other hand, when the thermal trough precedes the surface depression, there should be relative convergence.

c. $-\frac{V_T}{f}\frac{\partial f}{\partial s'}$ From this term one would get relative convergence when the thermal wind has a poleward component and relative divergence with an equatorward component. However, in mid-latitudes, the magnitude of the rate of change of the coriolis parameter is generally small compared to the other terms and for purposes of this investigation will be neglected.

It has been observed that divergence varies vertically and that the normal distribution over a surface low system is to have convergence in the lower layers with divergence above. In order to have development, the integrated divergence

aloft must over compensate for the integrated convergence in the lower layers. The amount of this over compensation should be a measure of the deepening or filling of the surface system. Instead of measuring the divergence and convergence for the entire vertical extent over a system, measuring it at some specified level or levels might be feasible and still give qualitatively similar results. Assuming a schematic vertical distribution of divergence over a cyclone or anticyclone as in Figure 1, Sutcliffe implies that the differences of divergence of any two levels should be an indication of the deepening or filling of the surface system.

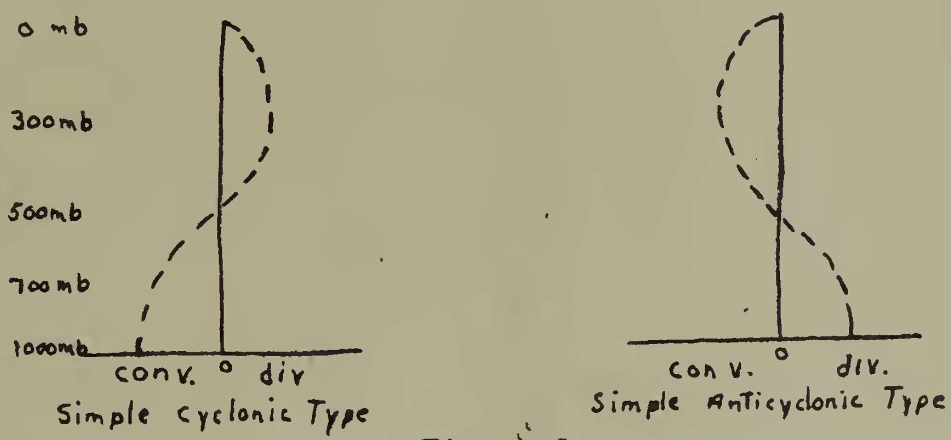


Figure 1

Schematic representation of the distribution of divergence.

That is, if the relative divergence over a cyclone is a large positive value, the deepening should be large and as this value decreases, the deepening should also decrease. An equivalent statement for anticyclones would associate negative values of relative divergence with anticyclonic development.

3. Investigation procedures and correlations.

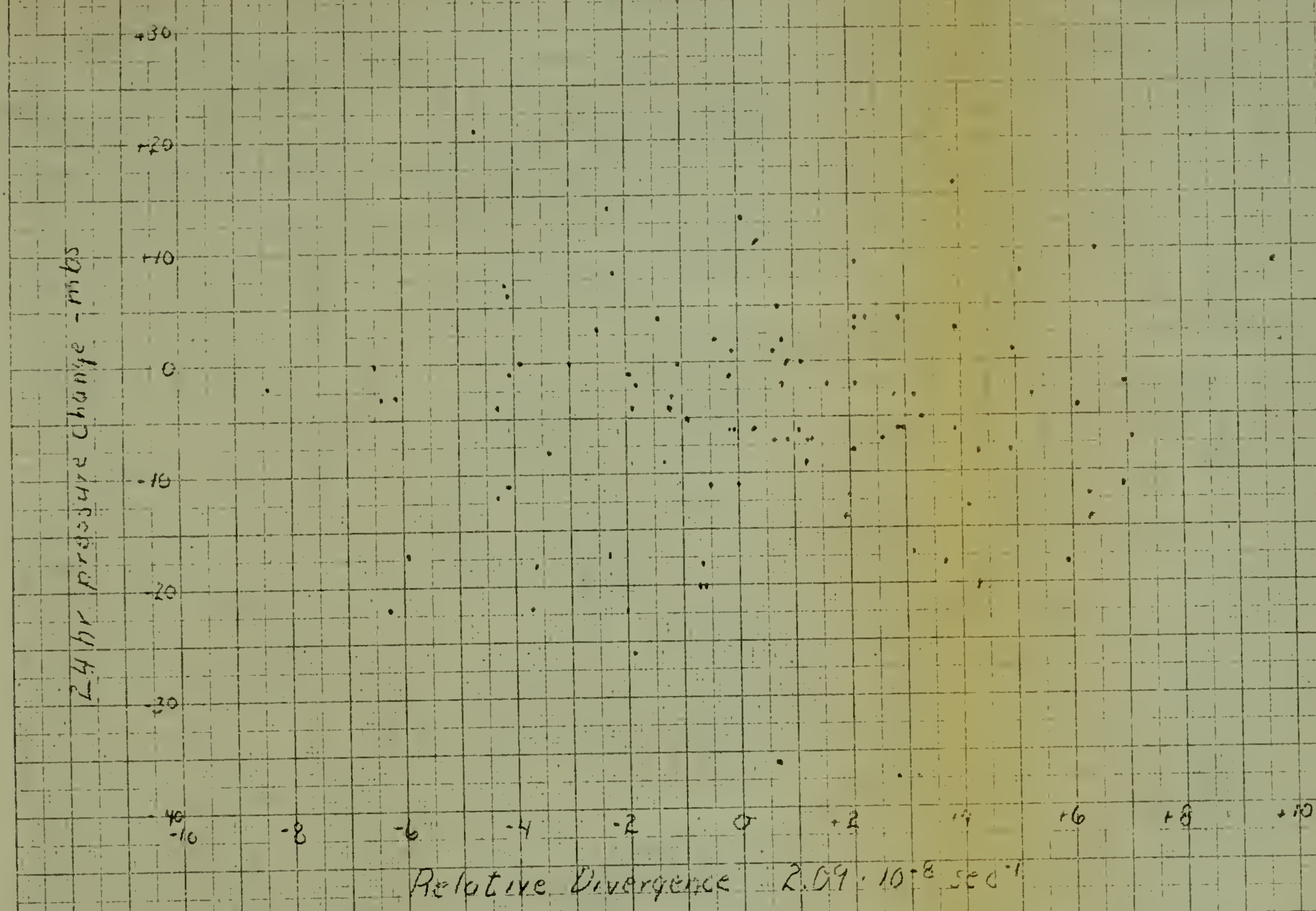
The purpose of this investigation was to see if it would be possible to correlate the present values of relative divergence as measured by terms (a) and (b) of equation (6) with a twenty-four hour deepening or filling of a surface low, as well as the average speed of movement over the same period of time. One hundred cases of surface lows in the period of December 1956 through February 1957 were used for the investigation. These lows were located in mid-latitudes, ranging from 28°N to 60°N. They were selected primarily over the North American continent in order to have the advantage of a dense reporting network, thereby reducing the analysis error due to subjectivity.

The initial attempt in the investigation was to evaluate the relative divergence at the center of a surface low. In regard to Sutcliffe's formula this meant evaluating the term $-\frac{V_T}{f}\frac{\partial \zeta_T}{\partial s'}$. The other term would have no contribution because the surface vorticity center and low center are normally coincident, thereby making $\frac{\partial \zeta_0}{\partial s'}$ equal to zero. Positions three hundred nautical miles upstream and downstream along the same thickness line were used and the calculation of thermal vorticity at these points was made by the finite difference approximation of $\frac{g}{f}\nabla^2 h$, namely, $\frac{4g}{f}\left(\frac{\bar{h}-h}{d^2}\right)$. A grid distance, d, of 5 degrees latitude was used which is consistent with the values used in graphical and numerical techniques. A second finite difference approximation was used to evaluate the rate of change of thermal

vorticity across the center. The latitude of the surface low center and the geostrophic thermal wind over the center were the other values used in the evaluation.

A plot of the relative divergence, arrived at by this method, versus the twenty-four hour pressure change is shown in Figure 2. There is no definite grouping of the values, which indicates that the future deepening or filling of the system cannot be predicted using the relative divergence at the center. A correlation coefficient of -.08 was computed from the data of this evaluation.

The results of the initial investigation were not conclusive so another approach was taken. In accordance with the kinematic formula of deepening or filling of the surface center should be proportional to the algebraic sum of the divergence ahead and behind the system [1]. That is, if there is more relative divergence ahead of the system than there is relative convergence behind the system, the system should deepen. To test this, it was necessary to evaluate terms (a) and (b) of Sutcliffe's development formula. The values of thermal vorticity and surface vorticity for positions along the same thickness line six hundred nautical miles ahead and behind the surface low center, as well as at the center were calculated. The geostrophic finite difference approximations of $\frac{g}{f}\nabla^2 h$ and $\frac{g}{f}\nabla^2 z_0$ were used. The finite difference approximation was used to compute the rate of change of these quantities along the thickness lines for points three hundred nautical miles ahead and behind the

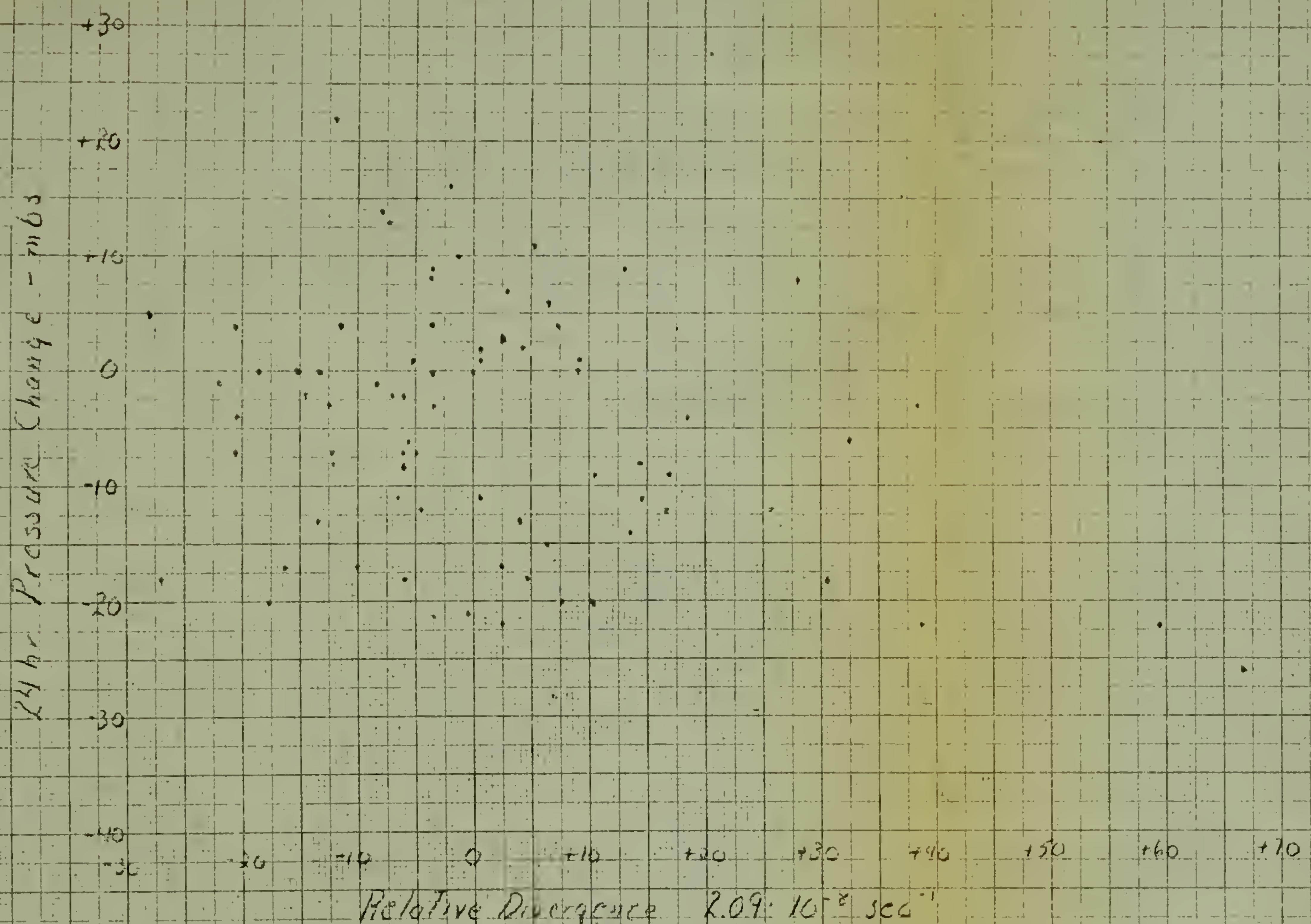


Scatter Diagram of Relative Divergence at Center and 24 hour Pressure Change

Figure No 2

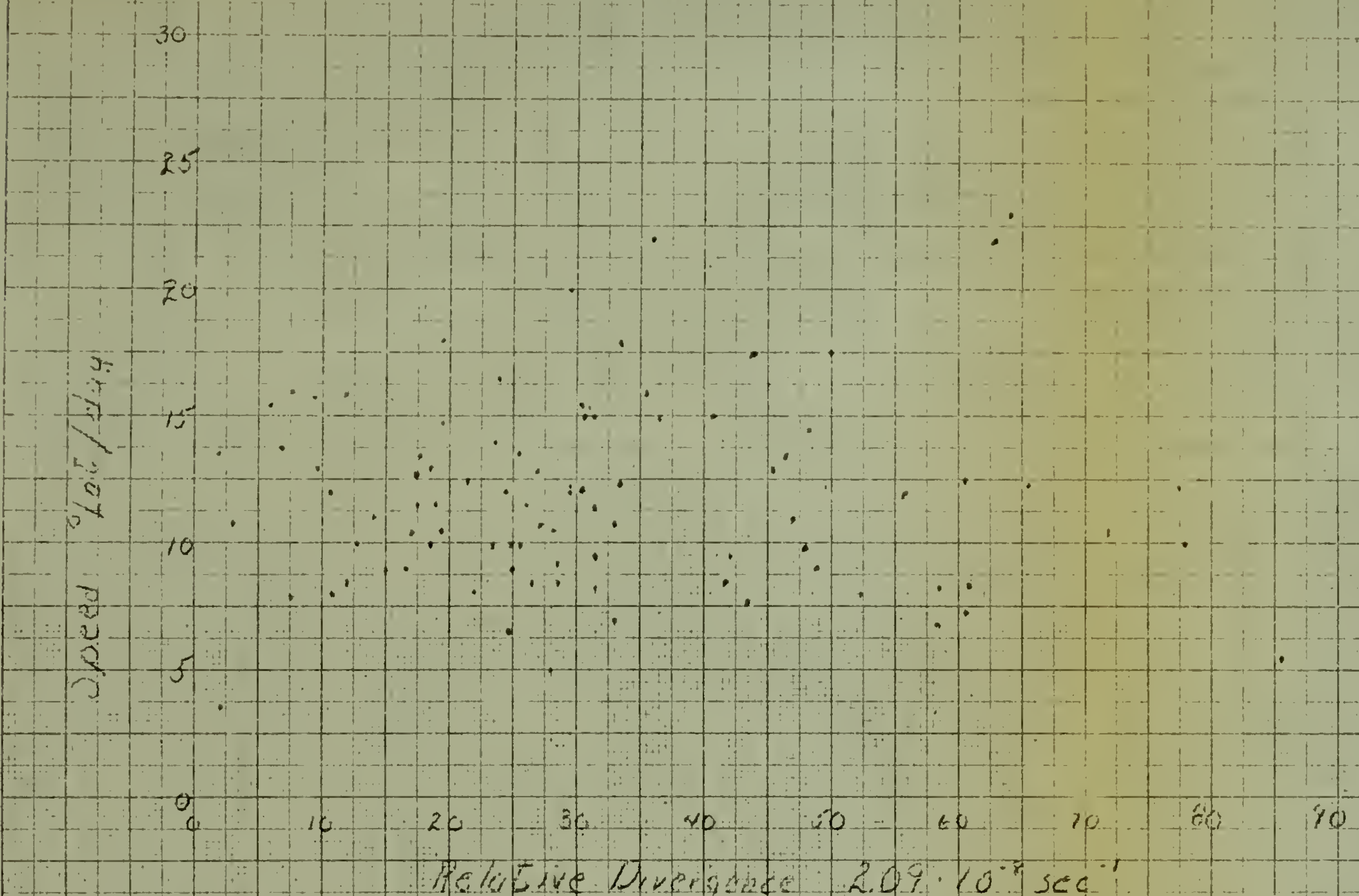
center. The thermal wind and latitude of the points three hundred nautical miles ahead and behind the surface center were the other values used in the computation. The algebraic sum of terms (a) and (b) were computed across the low and plotted versus the actual deepening or filling for twenty-four hours. Figure 3 shows this plot and the scatter of this diagram implies that very little can be inferred from the present values of relative divergence sums and the subsequent pressure change. A correlation coefficient of .28 between the relative divergence sums and the twenty-four hour pressure change was computed from the data in this test.

The third phase of the investigation consisted of an attempt to correlate the relative divergence with the twenty-four hour speed of the surface low system. The speed of the system should be proportional to the gradient of relative divergence across the system. To evaluate this quantity, the algebraic difference of the relative divergence values for the points three hundred nautical miles ahead and behind was taken. The subsequent speed of the associated low was evaluated in units of degrees of latitude per twenty-four hours. A scatter diagram of the speed versus the difference of relative divergence is shown in Figure 4. The correlation coefficient of these two quantities is -.02, and thus no definite statement can be made as to what average speed can be expected from the value of the difference of relative divergence across the system.



Scatter Diagram of Sum of Relative Divergence Across Center and 24 hr Pressure Change

Figure 3



Scatter Diagram of Difference of Relative Divergence Across Center and 24 hour Speed

Figure 4

4. Results and interpretations.

As a result of the observations and computations made in this investigation, it appears that on the basis of Sutcliffe's development formula the future development or displacement cannot be forecast.

The poor relationships resulting from the various phases of the investigation could be attributed to the fact that the actual vertical distribution of divergence, as well as relative divergence is more complex than that which was assumed by Sutcliffe. Also some of the assumptions involved in the derivation of the development formula are not completely realized at all times. Perhaps the length of time that was chosen for the future intensity was too long, but on a cursory examination of the twelve-hour pressure changes and speed, no improvement was apparent. It also may be indicated that the distances used in the evaluation may be too large and that positions closer to the center would give better approximations for the finite difference method. Perhaps making some modifications in the method of evaluation, or by empirically allowing for the neglected terms of the development formula, a more promising result could be obtained. An evaluation of the terms of Sutcliffe's development formula on a synoptic chart by use of a small grid system and Taylor Series expansion has been carried out by Sawyer and Matthewman [2], but no attempt was indicated of correlating the results with the future intensity of the cyclonic system.

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